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Technical Note TN-20

GEONICS EM39 BOREHOLE CONDUCTIVITY METER

THEORY OF OPERATION

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GEONICS LIMITED

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THEORY - EM39 BOREHOLE CONDUCTIVITY METER

INTRODUCTION

The electromagnetic induction method of measuring ground conductivity (described more fully in Geonics Limited Technical Note TN-6) is now widely used for mapping ground water contamination plumes, for ground water exploration, and for general geological mapping. A logical extension to the surface geophysical method was the development of a borehole conductivity measuring probe. Although such devices (called induction loggers) have been used by the oil industry for many years their sondes are generally unsuitable for geotechnical application.

For geotechnical work the principal requirements are:

(1) slim probes, since the boreholes are often only 5 cm in diameter (usually PVC cased).

(2) excellent vertical resolution so that, for example, thin contaminant plumes can be accurately located and resolved.

(3) high sensitivity combined with low noise and drift for measurements in resistive environments and also for detecting the small reductions in conductivity that are expected to occur when passing through contaminant plumes of organic liquids.

(4) excellent long term stability so that repeated measurements in a borehole (for monitoring purposes) will detect the very small variations in conductivity with time that result from small changes in the nature and extent of the contaminant plume.

(5) accurate measurement of the inphase component of the received signal for measurement of the terrain magnetic susceptibility and also for the detection of nearby buried metal objects such as scrap or metallic chemical containers.

(6) moderate depth of exploration, often only a few tens of meters and seldom greater than 200 meters.

In the design of an induction logger there is always a compromise between achieving a large lateral range of exploration into the host rock or soil on the one hand (combined with a small response from the borehole fluid itself) and a high degree of vertical resolution on the other. The first requirement is satisfied by large intercoil (Tx/Rx) spacing and the second by small spacing. Additional coils can be used to "focus" the array so as to reduce sensitivity to borehole fluid and improve vertical resolution. Based on the considerations outlined above an intercoil spacing of 50 cm was selected for the EM39 and a degree of focussing was employed. This technical note, in describing the theory of induction logging, illustrates the degree of success that has been met by the design in achieving good vertical resolution, reasonable lateral range of exploration, and insensitivity to conductive borehole fluid. Note that in this discussion distances are normalized with respect to ℓ, the Tx/Rx intercoil spacing, which, as mentioned above for the EM39 is 50 cm. Such normalized distances will be given in capital letters to emphasize this fact, thus R is the actual radial distance divided by 50 cm, etc.

APPARENT CONDUCTIVITY

Consider the geometry shown in Figure 1a,b in which a coaxial two-coil electromagnetic probe is employed to measure the electrical conductivity of the surrounding medium using the "low induction number" approximation. It has been shown by Doll that, under this approximation, and when all of the surrounding material has conductivity σ, the ratio of the emf in the receiver coil arising form the currents induced in the surrounding material to the emf caused directly by the transmitter is

$$\frac{V_s}{V_p} = \frac{j\omega\mu\sigma\ell^2}{2} \qquad (1)$$

where
$\omega = 2\pi f$
f = operating frequency (Hz)
μ = permeability of free space ($4\pi \times 10^{-7}$ h/m)
σ = ground conductivity (s/m)
ℓ = intercoil spacing (m)
$j = \sqrt{-1}$

It is useful to invert this equation so that the conductivity is indicated directly in terms of the measured voltage ratio

$$\sigma = \frac{2}{\omega\mu\ell^2} \left| \frac{V_S}{V_P} \right| \text{quad. component} \qquad (2)$$

In the event that the ground conductivity is not uniform, equation (2) defines an apparent conductivity σ_a which is now a function of the conductivity distribution.

If the conductivity distribution exhibits circular symmetry about the Tx/Rx axis, i.e., $\sigma = \sigma(R,Z)$ it becomes a simple matter to calculate the apparent conductivity since for this case the current in the surrounding material consists of horizontal loops centered on the Tx/Rx axis, as indicated in the figure. Furthermore under the "low induction number" approximation the magnitude of the current in a given loop is independent of the current in all other loops, being a function only of the frequency, the primary flux through the loop, and the (uniform) local ground conductivity for that loop.

In this case equation (1) becomes

$$\frac{V_S}{V_P} = \frac{j\omega\mu\ell^2}{2} \int_{-\infty}^{\infty}\int_{0}^{\infty} \sigma(R,Z)\ g(R,Z)\ dR\ dZ \qquad (3)$$

where $g(R,Z)$ is a weighting function which gives the relative contribution to the measured signal arising from the elemental current loop passing through R,Z. We can define the apparent conductivity in terms of this function as

$$\sigma_a = \int_{-\infty}^{\infty}\int_{0}^{\infty} g(R,Z) \;\; \sigma(R,Z) \; dR \; dZ \qquad (4)$$

The function g(R,Z) is normalized so that the integral over all space is equal to unity.

RADIAL SENSITIVITY

For a given intercoil spacing ℓ it is desirable to visualise how the sensitivity of the measurement varies as a function of radial distance R from the axis of the borehole; for example how far from the borehole axis does the measurement "see". To determine this we calculate the function $\phi(R)$, given by

$$\phi(R) = \int_{-\infty}^{\infty} g(R,Z) \; dZ \qquad (5)$$

This function defines the relative response to the measurement from the material in a thin-walled cylinder of radius R and wall thickness dR centered on the borehole axis. It depends on the number and configuration of receiver coils in the sonde. For a simple two-coil system (one transmitter, one receiver) curve (1) in Figure 2 shows the relative response with radial distance. We see that the sensitivity initially increases linearly with distance, peaks at about R = 0.45, and then decreases slowly with further increase in radial distance. The function shown in this figure is exactly analogous to the function $\phi(Z)$ calculated for the Geonics EM31 and EM34-3 and discussed in detail in Geonics Limited Technical Note TN-6. Unfortunately, for the coaxial borehole coil configuration the algebraic expression for $\phi(R)$ is not as simple as that for $\phi(Z)$.

Most induction loggers employ some sort of focussing (i.e., the addition of one or more additional receiver coils) to reduce the sensitivity to borehole fluid, to improve the vertical resolution, and also to partially or totally cancel out the primary field arising directly from the transmitter. The EM39 is no exception, and the resultant function $\phi_N(R)$ is shown as curve (2) in Figure 2, from which we observe that the sensitivity to material very near the borehole axis has been reduced by over an order of magnitude and that the peak response now occurs at R = 0.56 (or r = 28 cm for the EM39), a distance well away from the hole and into the surrounding material.

Another very useful function is obtained by taking the integral

$$R\ (R) = \int_R^{\infty}\int_{-\infty}^{\infty} g(R,Z)\ dZ\ dR = \int_R^{\infty} \phi(R)\ dR \qquad (6)$$

which gives the relative contribution to the instrument reading from all material at a radial distance greater than R. This function, again analogous to the function R(Z) discussed in TN-6, is plotted in Figure 3 as a function of R for the focussed array of the EM39. It shows, for example, that 50% of the instrument response arises from material at a radial distance greater than 1.16 intercoil spacings (58 cm for the EM39).

An important use for this function is that it enables us to easily calculate the apparent conductivity that the instrument will indicate when the ground conductivity varies radially only. For example, if the conductivity is given by

$$\begin{array}{ll} \sigma_1 \text{ for} & o < R < R_1 \\ \sigma_2 \text{ for} & R_1 < R < R_2 \\ \quad\vdots & \\ \sigma_n \text{ for} & R > R_{n-1} \end{array}$$

then the apparent conductivity read by the instrument will be given by

$$\begin{aligned} \sigma_a = & \ \sigma_1\ [1-R(R_1)] \qquad (7) \\ & + \sigma_2\ [R(R_1) - R(R_2)] \\ & + \ .... \\ & + \sigma_n\ \ R(R_{n-1}) \end{aligned}$$

Use of this simple expression will be made in the next section when dealing with the influence of the relatively conductive borehole fluid on the measurement.

SENSITIVITY TO BOREHOLE FLUID

In general we wish to measure the conductivity of the material at some remote distance from the borehole axis, however the measurement will also be influenced by the presence of the conductive fluid within the borehole itself. Consider the geometry shown in Figure 4 where a borehole of radius a, filled with water of conductivity σ_b, has been drilled into the ground which is saturated with water of the same conductivity. The relationship between the ground conductivity σ_g and the conductivity of the borehole water σ_b can usually be described by Archie's Law, which is

$$\sigma_g = \sigma_b \, n^m \tag{8}$$

In this equation n is the fractional porosity of the surrounding material (defined as the ratio of the volume of the fluid in the material to the total volume) and is always less than unity; m is a parameter between 1.2 and 1.8 which depends on the soil or rock type. (For a further treatment of the factors affecting the electrical conductivity of the ground refer to Geonics Limited Technical Note TN-5.) This equation shows that the conductivity of the borehole water is greater than that of the surrounding material, perhaps by a factor of ten or larger.

To evaluate the contribution from the borehole fluid to the measured conductivity use is made of equation (7) which becomes, for the concentric geometry illustrated in Figure 4.

$$\sigma_a = \sigma_b \left[1-R\left(\frac{a}{\ell}\right)\right] + \sigma_g R\left(\frac{a}{\ell}\right) \tag{9}$$

Let us assume that the largest borehole to be measured has a diameter of 6", whence $a/\ell = 0.15$. Examination of Figure 5 shows that, for $a/\ell \cong 0.2$, R(R) = 1.00 and thus, even for a borehole fluid with conductivity one hundred times greater than the host rock conductivity, the contribution from the borehole fluid will be negligible.

VERTICAL RESOLUTION

In the same way that we wish to know the radial sensitivity of the measurement we also wish to determine the resolution to horizontal layers of different conductivity and thickness. To determine this we calculate the function

$$T(Z) = \int_0^\infty g\ (R,Z)\ dR \qquad (10)$$

which gives the relative sensitivity to a very thin horizontal layer of thickness dZ (much less than the intercoil spacing) located at distance Z away from the center of the Tx/Rx system.

This function, for both a simple two-coil system and the focussed array of the EM39 is shown in Figure 5. We see that we have paid a slight premium for the reduced sensitivity to conductive borehole fluid in that the peak response to an infinitely thin horizontal sheet is assymetrical for the focussed array. That this is not usually a serious problem will be shown shortly. We note that the "full width to half maximum" response is about Z = 1.3 or 65 cm for the EM39, which gives us an estimate of the resolving power of the system to measure the thickness of the layers in horizontally layered structure.

Finally it is also useful to define an analogous function to equation (6) for calculations involving a horizontally layered earth. This becomes

$$H(Z) = \int_Z^\infty \int_0^\infty g\ (R,Z)\ dR\ dZ = \int_Z^\infty T(Z)\ dZ \qquad (11)$$

Equation (11) forms the basis of a simple computer program supplied with the EM39 which calculates the response (apparent conductivity) as this instrument passes downwards through an arbitrarily layered earth. The response calculated with the program for a four layered earth is shown in Figure 6. The low reading at the surface (approximately one-half of the actual conductivity of the upper layer) is caused by the fact that the sensor is not completely immersed in the earth. As the sensor travels downward the apparent conductivity rises to the to the correct value (to within 7%) at a depth of one meter. The rounding off of the edges of the profile is, of course, due to the finite vertical resolution of the instrument and the fact that the measured response is not quite the true response in the resistive and conductive layering arises from the finite "wings" of the response.

As the thickness of a layer decreases it becomes less than the resolution limit of the instrument and the shape of the response becomes essentially that of the instrument, as shown in Figure 7a,b. The peak value of the apparent conductivity becomes a function of the product of the conductivity (relative to the background) multiplied by the thickness of the layer. This feature is shown in Figure 7, where we see that two layers, one of conductivity contrast x thickness of 75 mS/m x 0.50 m, and the other of 150 mS.m x 0.25 m give peak anomalies of 34 and 42 mS/m respectively over the background. Both layers have a value of $S = \Delta\sigma t$ = 37.5 mS/m. It can be shown that for thin layers (where $t < \ell$) the peak anomaly response above background for a two coil system with ℓ = 50 cm is numerically equal to S. The focussing of the EM39 causes a slight departure from this value (dependent on the thickness of the thin layer) as indicated in the figure. Note that given an EM39 instrumental noise level of much less than 1 mS/m, an anomaly of 2.5 mS/m will, in the absence of excessive geological noise, be easily detected. This could correspond to a layer of $\Delta\sigma$ = 10 mS/m material one-quarter meter thick. Note also that the location of this layer would be determined to within a small fraction of a meter.

The response of the instrument to an insulating layer (such as might arise from hydrocarbon contamination) is also of interest. Figure 8 a,b illustrates the response for an insulating layer of thickness 0.5 and 0.25 m. It is interesting that, as for the conducting layer, when the thickness of the insulator is less than the intercoil spacing the thickness can no longer be resolved but again the peak response is given approximately by $S = \Delta\sigma t$, which now results in a reduction of signal rather than enhancement. In Figure 8a the conductivity of the resistive layer is 25 mS/m, the thickness 0.5 m and the background conductivity 50 mS/m; thus $S = \Delta\sigma t$ = (25-50) x 0.5 = 12.5 mS and, as seen in the figure, the response does indeed drop by 12 mS/m in going over the insulator. Thus the

EM39 should be a very useful tool for searching for insulators as well as for conductors.

CONCLUSION

This technical note has outlined the theory of the EM39 Borehole Conductivity Meter with particular emphasis on the sensitivity of the measurement as a function of radial distance, sensitivity to the borehole fluid, and vertical resolution.

A sensitive, high resolution borehole logger will have a wide range of applications. Firstly, in the hazardous waste site investigation field it will be used to measure depth, thickness, and conductivity of contaminated layers to identify and map them. During and after reclamation procedures repeat measurements will greatly facilitate plume monitoring. Alternatively, along coastal regions and in regions with brine injection wells the logger will be used to quickly and accurately measure the extent of saline intrusion into fresh groundwater. Finally, other areas of application will include geological mapping, detection and measurement of permafrost, measurement of coal seam thickness and depth, water bearing formation thickness, and general water quality.

GENERAL REFERENCES

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Kaufman, A.A., (1965) - Theory of Induction Logging, Navka Press (Trans. by G.V. Keller).

Saito, A., (1982) - Theory and Application of Induction Logging for Civil Engineering. M. Sc. Thesis. Colorado School of Mines.

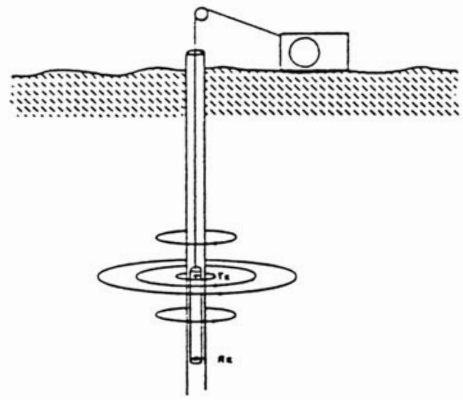


Fig. 1a Borehole Geometry

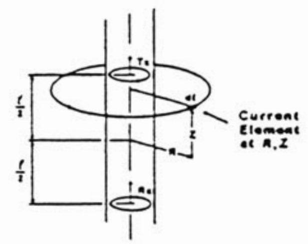
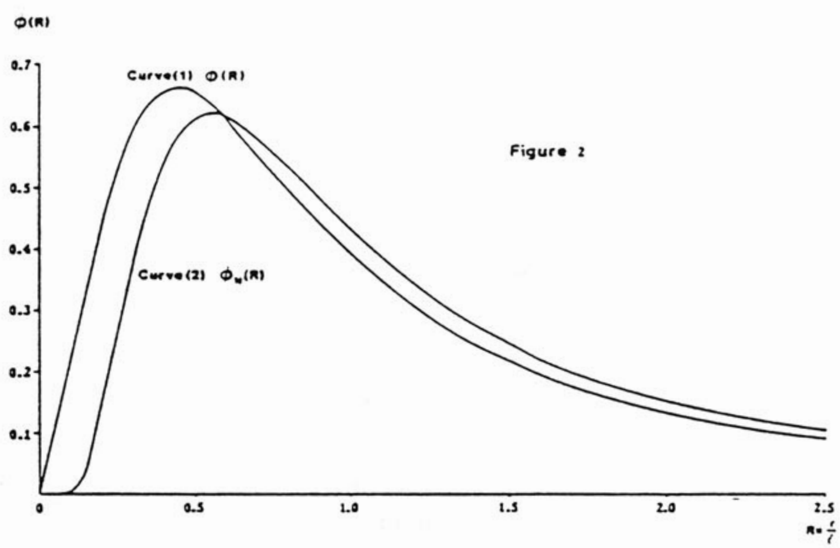
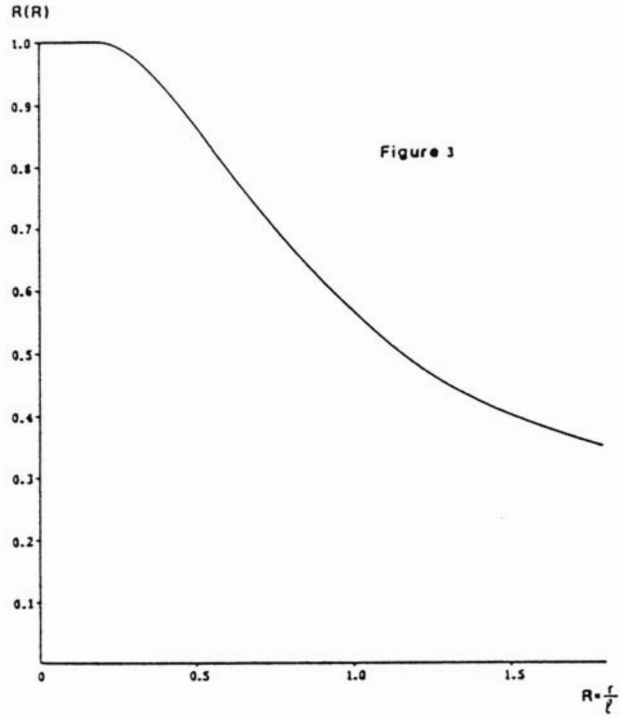


Fig. 1b Co-ordinate Geometry





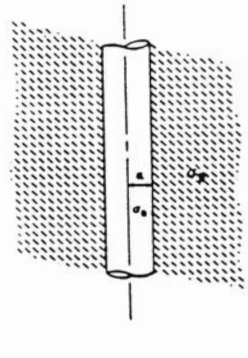


Fig. 4 Borehole Fluid Model

Figure 5

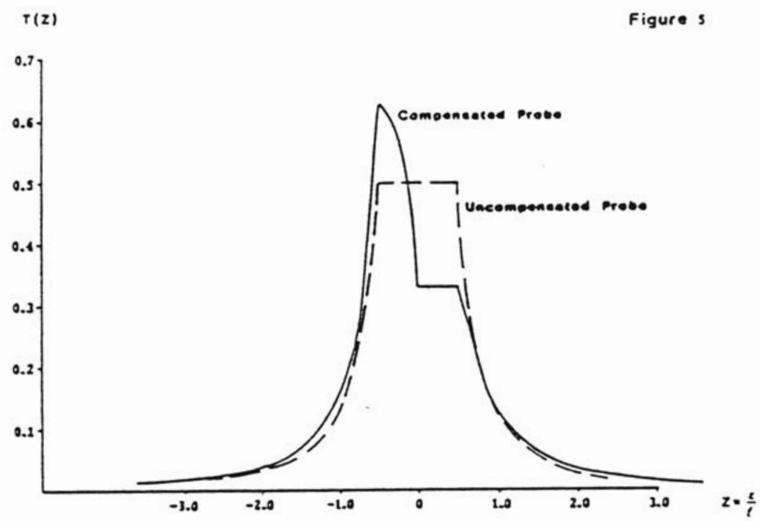
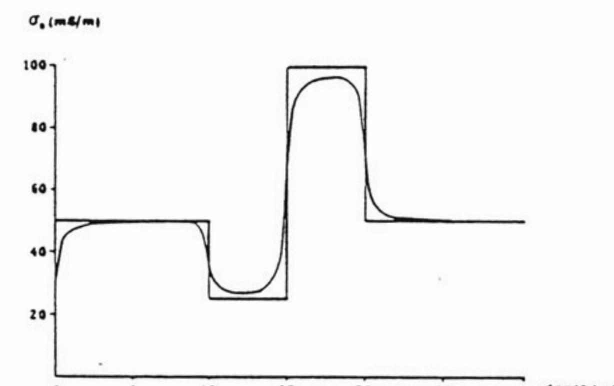
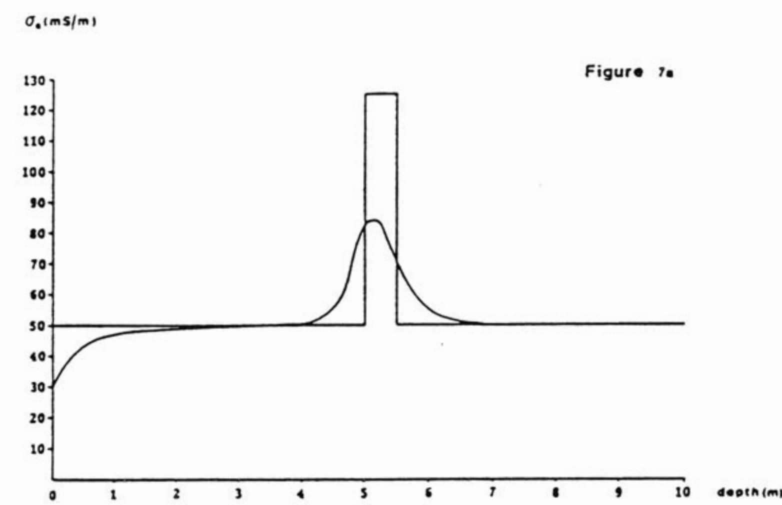


Figure 6




σa (mS/m)
Figure 7a
130
120
110
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
depth (m)

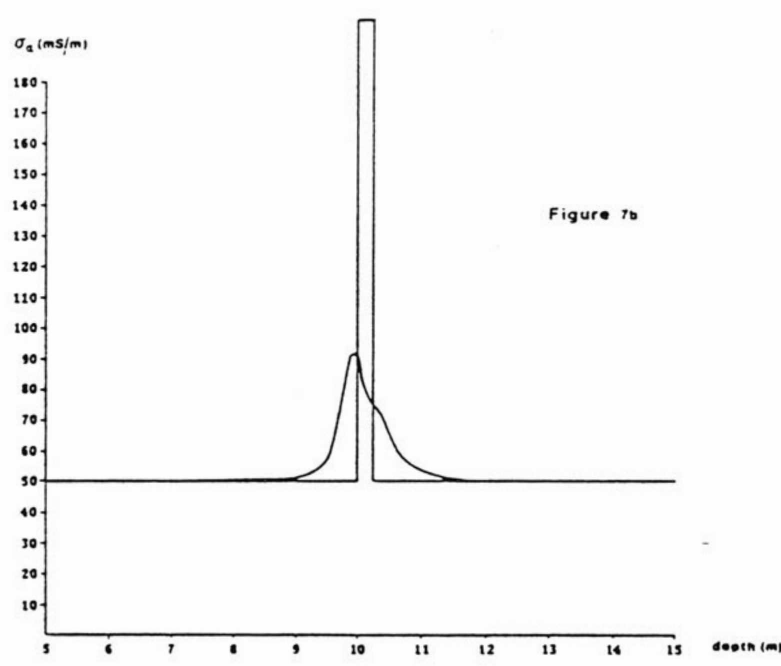

σa (mS/m)
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
Figure 7b
5
6
7
8
9
10
11
12
13
14
15
depth (m)

Figure 6a

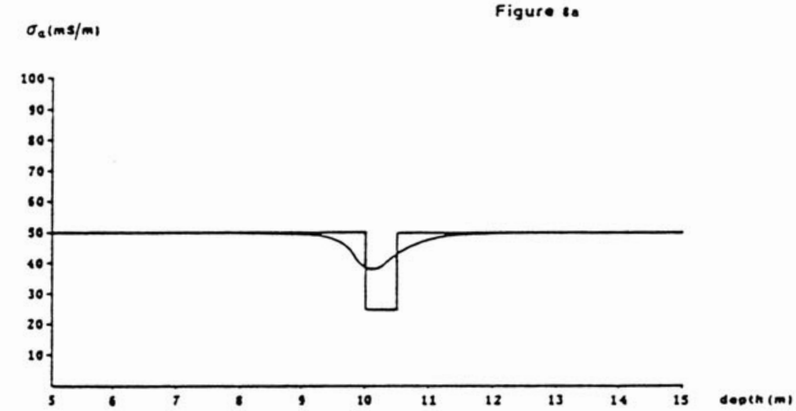
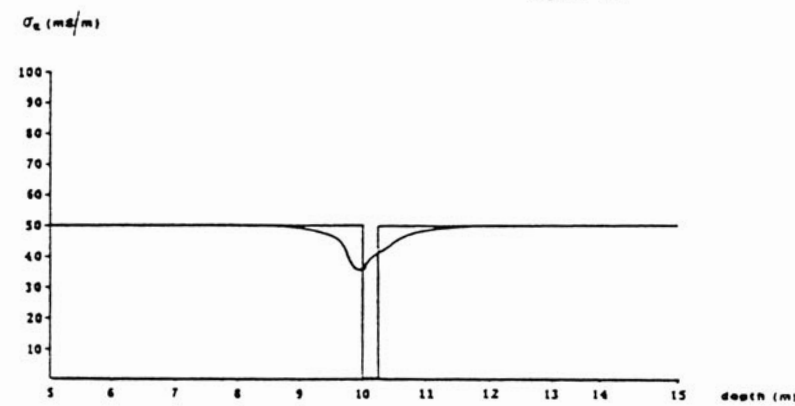


Figure 6b



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ADDENDUM

This addendum to Technical Note TN-20 discusses the deviation from linearity of the EM39 conductivity measurement caused by departure from the "low induction number" approximation at high values of terrain conductivity.

APPARENT CONDUCTIVITY

Kaufman and Keller show that the quadrature phase response of a simple two-coil (Tx/Rx) induction logger of intercoil spacing ℓ located in a homogeneous half-space of conductivity σ is given in the general case by

$$\frac{V_s}{V_p} = e^{-p} \left[(1+p) \sin p - p \cos p\right] \qquad \text{(A1)}$$

where $p = \ell/\delta$

and δ is the skin depth in the halfspace, given by

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \qquad \text{(A2)}$$

At low values of induction number, $p \ll 1$, and equation (A1) simplifies to

$$\frac{V_s}{V_p} \cong p^2 = \frac{\omega\mu\sigma\ell^2}{2} \qquad \text{(A3)}$$

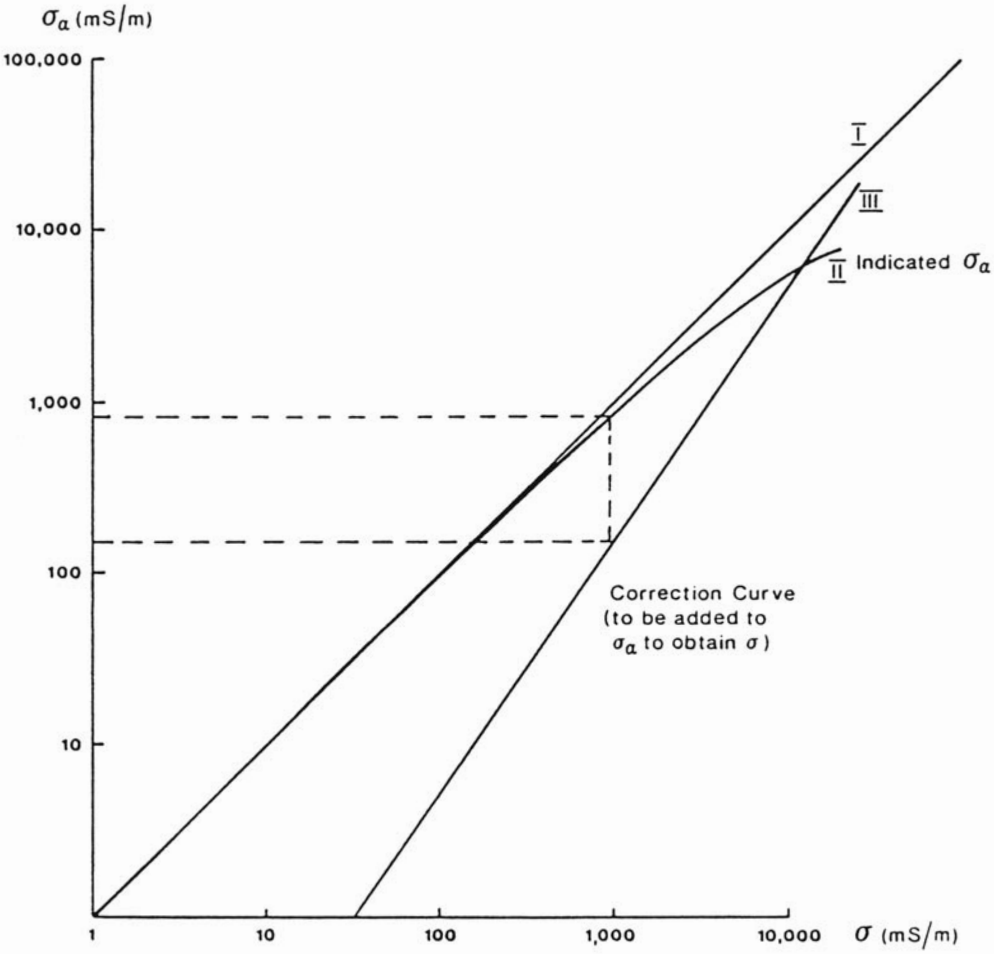
Equation (A3) is equation (1) of TN20. In the more general case of arbitrary value of induction number p, equation (A1) must be used to calculate the instrumental response as a function of conductivity; furthermore, for the EM39 the presence of the focussing coil must also be taken into account. The results of such calculations are shown in Figure A1, which shows three quantities plotted against the true homogeneous halfspace conductivity. These are:
Curve I - the apparent conductivity that would be read by the EM39 if the low induction number approximation was accurate over all values of homogeneous halfspace conductivity σ. In this case $\sigma_a = \sigma$.

Curve II - the apparent conductivity that will in fact be read by the EM39. We see that at low values of σ, $\sigma_a \cong \sigma$, but that at higher values of σ there will be a significant reduction in the apparent conductivity read by the EM39.

Curve III - the difference between the apparent conductivity read by the EM39 and the true conductivity. Note that curves II and III assume that the ground behaves as a homogeneous halfspace in the vicinity of the EM39 probe (i.e. exhibits no significant vertical or radial changes in conductivity within the region sensed by the EM39), which is often a reasonable assumption.

To use Curve III, simply read in the measured value of σ_a (for example 850 mS/m) on Curve II, then move vertically downwards to intersect Curve III to read the correction of 150 mS/m which must be added to 850 mS/m to give the true value of 1000 mS/m, as indicated on Curve I.

Kaufman, A.A., and Keller, G.V., Frequency and Transient Soundings. Elsevier Publishing Co. 1983.



EM39 CALIBRATION CURVE

Figure (1A)